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Entropy in Interval-Valued Fuzzy Soft Sets and an Application

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ABSTRACT

In this study, the classical entropy concept is extended to interval-valued fuzzy soft sets in order to objectively determine criterion weights by accurately capturing their discriminative power among a set of competing alternatives. In addition to entropy-based weighting, a novel correlation measure is formulated to enhance the reliability and robustness of the weighting process by considering interrelationships among criteria. Based on these theoretical developments, a systematic and structured decision-making algorithm is proposed. The applicability and effectiveness of the proposed approach are demonstrated through a comprehensive numerical example, which confirms that the method can efficiently handle uncertainty, vagueness, and incomplete information in complex multi-criteria decision-making environments.

Keywords: *Decision-making, Interval-valued Fuzzy Sets, Interval-valued Fuzzy Soft Sets*

a *0000-0003-1276-9004*

1 Introduction

Classical set theory operates with categories that have clearly defined boundaries; however, most of real-world and decision-making problems involve to uncertainty and incomplete information. Zadeh (1965) introduced fuzzy set theory [1], in which the membership of an element is expressed by a degree within the interval $[0,1]$. This formulation marked a significant advance in modeling imprecise information. Later, this concept was generalised into interval-valued fuzzy sets (IVFS) [2], where the membership degree is represented by a closed interval, $[\mu^{\nabla}(x), \mu^{\triangle}(x)]$. This extension provides a more expressive framework, capable of reflecting uncertainty arising from expert hesitation or data insufficiency. The basic algebraic operations for interval-valued fuzzy sets have been defined in the literature by Gorzałczany (1987) [3].

Soft set theory, proposed by Molodtsov (1999) [4], offers a parameterized structure designed to handle diverse forms of uncertainty within a unified framework. The combination of fuzzy and soft set theories led to the development of fuzzy soft sets and subsequently interval-valued fuzzy soft sets (IVFSS) [5]. These hybrid structures capture both parametric variability and membership uncertainty, more effective than classical uncertainty theories in many problems . By utilizing IVFSS, decision-makers can evaluate alternatives across different criteria in a more flexible and comprehensive manner.

Determining the relative importance, or weights, of criteria is a crucial component of the MCDM process. Weighting techniques are generally classified as subjective—based on expert opinion—or objective—derived from data. Among objective methods, entropy serves as a fundamental concept that quantifies the uncertainty or information content of a system. Firstly, introduced by Shannon [6] in information theory and later adapted to fuzzy environments, entropy measures the degree of distinction a criterion provides among alternatives. A lower entropy value indicates greater discriminative ability and, consequently, a higher weight. In the present study, this concept is extended to the interval-valued fuzzy soft set environment.

This study aims to develop a new entropy-based multi-criteria decision-making method within the framework of IVFSS. Building upon the generalization of entropy and correlation concepts to interval-valued soft sets, a theoretical foundation is established. Based on this framework, a systematic and fully objective decision-making algorithm is proposed, in which the criterion weights are derived directly from data. The practicality and effectiveness of the proposed approach are demonstrated through a detailed numerical example.

The main contributions of this study can be summarized as follows:

(i) A novel entropy measure is proposed for interval-valued fuzzy soft sets, enabling the quantification of uncertainty under both parameterization and interval-valued membership information.

(ii) A new correlation coefficient is introduced to evaluate the relationship between interval-valued fuzzy soft sets.

(iii) An objective weighting mechanism is developed by integrating entropy and correlation information.

(iv) A decision-making algorithm is constructed and applied to an interval-valued fuzzy soft decision environment.

2 Preliminaries

Definition 1 [2]Let X be a non-empty set, $\mu_{\tilde{A}}: X \to D[0,1]$, where $D[0,1]$ expresses the set of all closed subintervals of $[0,1]$. Then $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$ is called an interval valued fuzzy set in X.

All interval valued fuzzy sets on X are denoted by $IVFS(X)$. Here $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^{\nabla}(x), \mu_{\tilde{A}}^{\triangle}(x)]$ can be expressed by a pair of functions, where $\mu_{\tilde{A}}^{\nabla}(x) \le \mu_{\tilde{A}}^{\triangle}(x)$ for all $x \in X$, namely the lower and upper membership functions, respectively.

Definition 2 [3]Let X be a non-empty set and $\tilde{A}, \tilde{B} \in IVFS(X)$. Then the following operations are defined:

1) If for all $x \in X$, $\mu_{\tilde{A}}^{\nabla}(x) \le \mu_{\tilde{B}}^{\nabla}(x)$ and $\mu_{\tilde{A}}^{\triangle}(x) \le \mu_{\tilde{B}}^{\triangle}(x)$, then $\tilde{A}$ is called an interval-valued fuzzy subset of $\tilde{B}$ and is denoted by $\tilde{A} \le \tilde{B}$.

2) If $\mu_{\tilde{C}}^{\nabla}(x) = \min\{\mu_{\tilde{A}}^{\nabla}(x), \mu_{\tilde{B}}^{\nabla}(x)\}$ and $\mu_{\tilde{C}}^{\triangle}(x) = \min\{\mu_{\tilde{A}}^{\triangle}(x), \mu_{\tilde{B}}^{\triangle}(x)\}$for all $x \in X$, then $\tilde{C}$ is called the interval-valued fuzzy intersection of $\tilde{A}$ and $\tilde{B}$ and is denoted by $\tilde{C} = \tilde{A} \wedge \tilde{B}$.

3) If $\mu_{\tilde{D}}^{\nabla}(x) = \max\{\mu_{\tilde{A}}^{\nabla}(x), \mu_{\tilde{B}}^{\nabla}(x)\}$ and $\mu_{\tilde{D}}^{\triangle}(x) = \max\{\mu_{\tilde{A}}^{\triangle}(x), \mu_{\tilde{B}}^{\triangle}(x)\}$ for all $x \in X$, then $\tilde{D}$ is called the interval-valued fuzzy union of $\tilde{A}$ and $\tilde{B}$ and is denoted by $\tilde{D} = \tilde{A} \vee \tilde{B}$.

4) If $\mu_{\tilde{E}}^{\triangle}(x) = 1 - \mu_{\tilde{A}}^{\triangle}(x)$, $\mu_{\tilde{E}}^{\nabla}(x) = 1 - \mu_{\tilde{A}}^{\nabla}(x)$ for all $x \in X$, then $\tilde{E}$ is called the interval-valued fuzzy complement of $\tilde{A}$ and is denoted by $\tilde{E} = \tilde{A}^c$.

5) If $\mu_{\tilde{F}}(x) = [0,1]$ for all $x \in X$, then $\tilde{F}$ is called the interval-valued universal fuzzy set and is denoted by $\tilde{1}$.

6) If $\mu_{\tilde{G}}(x) = [0,0]$ for all $x \in X$, then $\tilde{G}$ is called the interval-valued empty fuzzy set and is denoted by $\tilde{0}$.

Definition 3 [4]Let X be a non-empty set, E set of parameters, $A \subseteq E$ and $F_A: A \to P(X)$ a function. Then the set $(F, A) = \{(e, F_A(e)): e \in E\}$ is called a soft set on X. Here the F_A function is called the approximation function.

Definition 4 [5]Let $A \subseteq E$ and $f_{\tilde{A}}: A \to IVFS(X)$ be a function. Then the set $(f_{\tilde{A}}, E) = \{(e, f_{\tilde{A}}(e)): e \in E, f_{\tilde{A}}(e) \in IVFS(X)\}$ is called an interval valued fuzzy soft set on X. Here, the function $f_{\widetilde{IA}}$ is called the approximation function of $(f_{\tilde{A}}, E)$.

Throughout the study, we will denote an $(f_{\tilde{A}}, E)$ interval valued fuzzy soft set with the approximation function $f_{\tilde{A}}$ for the sake of simplicity. The family of all interval valued fuzzy soft sets on X with parameter set E is denoted by $IVFS(X, E)$. In an $f_{\tilde{A}}$ interval valued fuzzy soft set, the elements whose image is empty may not be written. Therefore, the parameter set of all interval valued fuzzy soft sets on X can be considered as E.

Definition 5 [5]Let X be a non-empty set, E set of parameters. Then the basic operations of interval valued fuzzy soft sets are defined as follows;

1) Let $f_{\tilde{A}}(e), f_{\tilde{B}}(e) \in IVFS(X, E)$. If $f_{\tilde{A}}(e) \le f_{\tilde{B}}(e)$ for $e \in E$, then $f_{\tilde{A}}$ is called the interval valued fuzzy soft subset of $f_{\tilde{B}}$ and is denoted by $f_{\tilde{A}} \cong f_{\tilde{B}}$.

2) Let $f_{\tilde{A}}(e), f_{\tilde{B}}(e) \in IVFS(X, E)$. If $f_{\tilde{C}}(e) = f_{\tilde{A}}(e) \vee f_{\tilde{B}}(e)$ for all $e \in E$, then the interval valued fuzzy soft set $f_{\tilde{C}}$ is called interval valued fuzzy soft union of $f_{\tilde{A}}$ and $f_{\tilde{B}}$ is denoted by $f_{\tilde{C}} = f_{\tilde{A}} \,\widetilde{\cup}\, f_{\tilde{B}}$.

3) Let $f_{\tilde{A}}(e), f_{\tilde{B}}(e) \in IVFS(X, E)$. If $f_{\tilde{C}}(e) = f_{\tilde{A}}(e) \wedge f_{\tilde{B}}(e)$ for all $e \in E$, then the interval valued fuzzy soft set $f_{\tilde{C}}$ is called interval valued fuzzy soft intersection of $f_{\tilde{A}}$ and $f_{\tilde{B}}$ is denoted by $f_{\tilde{C}} = f_{\tilde{A}} \,\widetilde{\cap}\, f_{\tilde{B}}$.

4) Let $f_{\tilde{A}}(e) \in IVFS(X, E)$. If $f_{\tilde{A}}^{C}(e) = (f_{\tilde{A}}(e))^c$ for all $e \in E$,then the $f_{\tilde{A}}^{c}$ is called interval valued fuzzy soft complement of $f_{\tilde{A}}$.

5) If $f_{\tilde{A}}(e) = \tilde{0}$ for all $e \in E$, then the interval valued fuzzy soft set $f_{\tilde{A}}$ is called the empty interval valued fuzzy soft set and is denoted by $f_{\tilde{0}}$.

6) If $f_{\tilde{A}}(e) = \tilde{1}$ for all $e \in E$, then the interval valued fuzzy soft set $f_{\tilde{A}}$ is called the universal interval valued fuzzy soft set and is denoted by $f_{\tilde{X}}$.

Example 1 Let be $E = \{e_1, e_2, e_3\}$, $X = \{x_1, x_2, x_3\}$,

$f_{\tilde{A}} = \{(e_1, \{(x_1, [0.3,0.9]), (x_2, [0.2,0.7]), (x_3, [0.5,0.6])\}), (e_2, \{(x_1, [0,0.3]), (x_3, [0.3,1]\}), (e_3, \{(x_1, [0.2,0.8]), (x_2, [0.5,0.9])\})\}$

and

$f_{\tilde{B}} = \{(e_1, \{(x_1, [0.5,0.6]), (x_2, [0.2,0.8]), (x_3, [0.8,0.9])\}),$
$(e_2, \{(x_2, [0.2,0.9]), (x_3, [0.1,0.6]\}), (e_3, \{(x_1, [0.2,1]), (x_3, [0.8,1])\})\}.$

Then;

$f_{\tilde{A}} \,\widetilde{\cup}\, f_{\tilde{B}} = \{(e_1, \{(x_1, [0.5,0.9]), (x_2, [0.2,0.8]), (x_3, [0.8,0.9])\}), (e_2, \{(x_1, [0,0.3]), (x_2, [0.2,0.9]), (x_3, [0.3,1]\}),$
$(e_3, \{(x_1, [0.2,1]), (x_2, [0.5,0.9]) , (x_3, [0.8,1]\})\}$

$f_{\tilde{A}} \,\widetilde{\cap}\, f_{\tilde{B}} = \{(e_1, \{(x_1, [0.3,0.6]), (x_2, [0.2,0.7]), (x_3, [0.5,0.6])\}),$
$(e_2, \{(x_3, [0.1,0.6]\}), (e_3, \{(x_1, [0.2,0.8])\})\}$

$f_{\tilde{A}}^{c} = \{\{(e_1, \{(x_1, [0.1,0.7]), (x_2, [0.3,0.8]), (x_3, [0.4,0.5])\}),$
$(e_2, \{(x_1, [0.7,1]), \{(x_2, [1,1]), (x_3, [0,0.7]\}), (e_3, \{(x_1, [0.2,0.8]), (x_2, [0.1,0.5]), (x_3, [1,1])\})\}$

In [7], Burillo and Bustince introduced several definitions of intuitionistic entropy for intuitionistic fuzzy sets. Building upon their work, we extend these concepts to establish the notions of entropy and correlation within the framework of interval-valued fuzzy soft sets.

Definition 6 *Let* $E=\{e_1,e_2,\dots,e_m\}$, $X=\{x_1,x_2,\dots x_n\}$ *and* $f_{\tilde{A}}$, $f_{\tilde{B}}$ *be two interval valued fuzzy soft sets on* $IVFS(X,E)$. *The correlation, informational interval valued energies and correlation coefficients of* $f_{\tilde{A}}$ *and* $f_{\tilde{B}}$ *are* $C(f_{\tilde{A}},f_{\tilde{B}})$, $T(f_{\tilde{A}})$, $T(f_{\tilde{B}})$ *and* $k(f_{\tilde{A}},f_{\tilde{B}})$ *, respectively, and are defined in the form*

$$\mu^{\triangle}_{f_{\tilde{A}}Avg}=\frac{1}{nm}(\sum_{i=1}^{n}\sum_{j=1}^{m}\mu^{\triangle}_{f_{\tilde{A}}(e_i)}(c_j))$$

$$\mu^{\triangle}_{f_{\tilde{B}}Avg}=\frac{1}{nm}(\sum_{i=1}^{n}\sum_{j=1}^{m}\mu^{\triangle}_{f_{\tilde{B}}(e_i)}(c_j))$$

$$\mu^{\triangledown}_{f_{\tilde{A}}Avg}=\frac{1}{nm}(\sum_{i=1}^{n}\sum_{j=1}^{m}\mu^{\triangledown}_{f_{\tilde{A}}(e_i)}(c_j))$$

$$\mu^{\triangledown}_{f_{\tilde{B}}Avg}=\frac{1}{nm}(\sum_{i=1}^{n}\sum_{j=1}^{m}\mu^{\triangledown}_{f_{\tilde{B}}(e_i)}(c_j))$$

$$C^{\triangle}(f_{\tilde{A}},f_{\tilde{B}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})(\mu^{\triangle}_{f_B(e_i)}(c_j)-\mu^{\triangle}_{f_BAvg})$$

$$C^{\triangledown}(f_{\tilde{A}},f_{\tilde{B}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangledown}_{f_A(e_i)}(c_j)-\mu^{\triangledown}_{f_AAvg})(\mu^{\triangledown}_{f_B(e_i)}(c_j)-\mu^{\triangledown}_{f_BAvg})$$

$$C(f_{\tilde{A}},f_{\tilde{B}})=\frac{C^{\triangle}(f_{\tilde{A}},f_{\tilde{B}})+C^{\triangledown}(f_{\tilde{A}},f_{\tilde{B}})}{2}$$

$$T^{\triangle}(f_{\tilde{A}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})^2$$

$$T^{\triangledown}(f_{\tilde{A}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangledown}_{f_A(e_i)}(c_j)-\mu^{\triangledown}_{f_AAvg})^2$$

$$T^{\triangle}(f_{\tilde{B}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangledown}_{f_B(e_i)}(c_j)-\mu^{\triangledown}_{f_BAvg})^2$$

$$T^{\triangledown}(f_{\tilde{B}})=\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangledown}_{f_B(e_i)}(c_j)-\mu^{\triangledown}_{f_BAvg})^2$$

$$k^{\triangle}(f_{\tilde{A}},f_{\tilde{B}})=\frac{C^{\triangle}(f_{\tilde{A}},f_{\tilde{B}})}{\sqrt{T^{\triangle}(f_{\tilde{A}})T^{\triangle}(f_{\tilde{B}})}}$$

$$k^{\triangledown}(f_{\tilde{A}},f_{\tilde{B}})=\frac{C^{\triangledown}(f_{\tilde{A}},f_{\tilde{B}})}{\sqrt{T^{\triangledown}(f_{\tilde{A}})T^{\triangledown}(f_{\tilde{B}})}}$$

$$k(f_{\tilde{A}},f_{\tilde{B}})=\frac{k^{\triangle}(f_{\tilde{A}},f_{\tilde{B}})+k^{\triangledown}(f_{\tilde{A}},f_{\tilde{B}})}{2}$$

Theorem 1 *Let* $E=\{e_1,e_2,\dots,e_m\}$, $X=\{x_1,x_2,\dots x_n\}$ *and* $f_{\tilde{A}}$,$f_{\tilde{B}}$ *be two interval valued fuzzy soft sets on* $IVFS(X,E)$. *Then the following is true;*

$$-1\le k(f_{\tilde{A}},f_{\tilde{B}})\le 1$$

$$k(f_{\tilde{A}},f_{\tilde{B}})=k(f_{\tilde{B}},f_{\tilde{A}})$$

If $k(f_{\tilde{A}},f_{\tilde{B}})=1$, then $f_{\tilde{A}}=f_{\tilde{B}}$.

Proof.

Since for all $i\in\{1,2,\dots,n\}$ and $j\in\{1,2,\dots m\}$, by the Cauchy–Schwarz inequality, we have

$$\begin{aligned}(C^{\triangle}(f_{\tilde{A}},f_{\tilde{B}}))^2 &= (\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})(\mu^{\triangle}_{f_B(e_i)}(c_j)-\mu^{\triangle}_{f_BAvg}))^2\\ &= (\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})(\mu^{\triangle}_{f_B(e_i)}(c_j)-\mu^{\triangle}_{f_BAvg})).\\ &(\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})(\mu^{\triangle}_{f_B(e_i)}(c_j)-\mu^{\triangle}_{f_BAvg}))\\ &\le \sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j)-\mu^{\triangle}_{f_AAvg})^2.\\ &\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_B(e_i)}(c_j)-\mu^{\triangle}_{f_BAvg})^2\\ &= T(f_{\tilde{A}})T(f_{\tilde{B}})\end{aligned}$$

Therefore

$$k^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}}) = \frac{\left|C^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}})\right|}{\sqrt{T^{\triangle}(f_{\tilde{A}})T^{\triangle}(f_{\tilde{B}})}} \leq 1$$

Similarly

$$k^{\triangledown}(f_{\tilde{A}}, f_{\tilde{B}}) = \frac{\left|C^{\triangledown}(f_{\tilde{A}}, f_{\tilde{B}})\right|}{\sqrt{T^{\triangledown}(f_{\tilde{A}})T^{\triangledown}(f_{\tilde{B}})}} \leq 1$$

Consequently, this shows that $-1 \leq k(f_{\tilde{A}}, f_{\tilde{B}}) \leq 1$.
Considering the definitions, both $C^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}})C^{\triangledown}(f_{\tilde{A}}, f_{\tilde{B}})$, and the products $T^{\triangle}(f_{\tilde{A}}).T^{\triangle}(f_{\tilde{B}}), T^{\triangledown}(f_{\tilde{A}}).T^{\triangledown}(f_{\tilde{B}})$ are clearly symmetric. Therefore $k^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}})$ and $k^{\triangledown}(f_{\tilde{A}}, f_{\tilde{B}})$ are also symmetric. Consequently, it follows that $k(f_{\tilde{A}}, f_{\tilde{B}}) = k(f_{\tilde{B}}, f_{\tilde{A}})$.

Firstly, the fact that $k = 1$ implies, by the definiton, that both $k^{\triangle} = 1$ and $k^{\triangledown} = 1$. Then we have $\left|C^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}})\right| = T^{\triangle}(f_{\tilde{A}})T^{\triangle}(f_{\tilde{B}})$ and $\left|C^{\triangledown}(f_{\tilde{A}}, f_{\tilde{B}})\right| = T^{\triangledown}(f_{\tilde{A}})T^{\triangledown}(f_{\tilde{B}})$. If $\left|C^{\triangle}(f_{\tilde{A}}, f_{\tilde{B}})\right| = T^{\triangle}(f_{\tilde{A}})T^{\triangle}(f_{\tilde{B}})$, then

$$(\sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j) - \mu^{\triangle}_{f_A Avg})(\mu^{\triangle}_{f_B(e_i)}(c_j) - \mu^{\triangle}_{f_B Avg}))^2 = \sum_{i=1}^{n}\sum_{j=1}^{m}(\mu^{\triangle}_{f_A(e_i)}(c_j) - \mu^{\triangle}_{f_A Avg})^2$$

This implies that $\mu^{\triangle}_{f_A(e_i)}(c_j) - \mu^{\triangle}_{f_A Avg} = \mu^{\triangle}_{f_B(e_i)}(c_j) - \mu^{\triangle}_{f_B Avg}$, for all $i \in \{1,2,..m\}, j \in \{1,2,..n\}$. Hence, this equality holds if and only if $\mu^{\triangle}_{f_A(e_i)}(c_j) = \mu^{\triangle}_{f_B(e_i)}(c_j)$, for all $i \in \{1,2,..m\}, j \in \{1,2,..n\}$. Similarly, it follows that $\mu^{\triangledown}_{f_A(e_i)}(c_j) = \mu^{\triangledown}_{f_B(e_i)}(c_j)$, for all $i \in \{1,2,..m\}, j \in \{1,2,..n\}$. This clearly shows that $f_{\tilde{A}} = f_{\tilde{B}}$.

In order to quantify the degree of uncertainty inherent in interval-valued fuzzy soft sets, we introduce an entropy measure analogous to the Shannon entropy [6].

Definition 7 *Let $E = \{e_1, e_2, \ldots, e_m\}$, $X = \{x_1, x_2, \ldots x_n\}$ and $f_{\tilde{A}}$ be an interval valued fuzzy soft set on $IVFS(X, E)$. Then the number $E(f_{\tilde{A}})$ is called the interval valued fuzzy soft entropy and is defined in the form;*

$$E(f_{\tilde{A}}) = \frac{1}{m}\sum_{i=1}^{m} E_i.$$

where

$$p^{\triangle}_{ij} = \frac{\mu^{\triangle}_{f_A(e_i)}(x_j)}{\sum_{k=1}^{n}\mu^{\triangle}_{f_A(e_i)}(x_k)}$$

$$p^{\triangledown}_{ij} = \frac{\mu^{\triangledown}_{f_A(e_i)}(x_j)}{\sum_{k=1}^{n}\mu^{\triangledown}_{f_A(e_i)}(x_k)}$$

$$E_i = \frac{-1}{2\ln n}\sum_{j=1}^{n}(p^{\triangle}_{ij}\ln(p^{\triangle}_{ij}) + p^{\triangledown}_{ij}\ln(p^{\triangledown}_{ij})).$$

Here, $\mu^{\triangle}_{f_A(e_i)}(x_j)$ and $\mu^{\triangledown}_{f_A(e_i)}(x_j)$ denote, respectively, the upper and lower membership degrees of the element $x_j \in X$ corresponding to the parameter $e \in E$.

The proposed entropy reflects the uncertainty contained in interval-valued fuzzy soft information. Smaller entropy values indicate higher discrimination among alternatives, whereas larger entropy values imply greater uncertainty.

3. Decision-Making Algorithm Based on Entropy Weight

In this section, we present a method for handling multi-criteria decision-making (MCDM) problems under uncertainty, based on the framework of interval-valued fuzzy soft sets. The proposed algorithm is designed to address situations in which the criterion weights are not known in advance. In this approach, the entropy measure defined for interval-valued fuzzy soft sets is utilized to derive the criterion weights, which play a crucial role in determining the final decision set.

Algorithm

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives and $E = \{e_1, e_2, \ldots, e_m\}$ be a set of criteria. The evaluation of each alternative x_j with respect to each criterion e_i is given by an interval, representing the degree to which alternative x_j satisfies criterion e_i. This information is encapsulated in an interval-valued fuzzy soft set $f_{\tilde{A}}$.

Step 1: Calculate the Entropy for Each Criterion

Compute the entropy E_i for each criterion e_i:

$$E_i = -\frac{1}{2\ln n}\sum_{j=1}^{n}\left(p^{\triangle}_{ij}\ln(p^{\triangle}_{ij}) + p^{\triangledown}_{ij}\ln(p^{\triangledown}_{ij})\right)$$

where

$$p^{\triangle}_{ij} = \frac{\mu^{\triangle}_{f_A(e_i)}(x_j)}{\sum_{k=1}^{n}\mu^{\triangle}_{f_A(e_i)}(x_k)}, p^{\triangledown}_{ij} = \frac{\mu^{\triangledown}_{f_A(e_i)}(x_j)}{\sum_{k=1}^{n}\mu^{\triangledown}_{f_A(e_i)}(x_k)}$$

Step 2: Compute the Weight of Each Criterion

Obtain the weight w_i for each criterion by normalizing the degrees of divergence:

$$w_i = \frac{1-E_i}{m-E(f_{\tilde{A}})}$$

Step 3: Construct the Weighted Normalized Decision Matrix

Multiply the normalized decision matrix by the corresponding criterion weights:

$$W_j = \sum_{i=1}^{m} w_i \cdot \frac{\mu^{\triangle}_{f_A(e_i)}(x_j)+\mu^{\triangledown}_{f_A(e_i)}(x_j)}{2}$$

Step 4: Rank the Alternatives

Rank the alternatives in descending order of W_j. The alternative with the highest W_j value is the best choice.

Computationally, the proposed method requires entropy calculations for each criterion and weighted aggregation across alternatives. For a decision matrix containing m alternatives and n criteria, the computational complexity is O(mn), which is comparable to conventional entropy-based weighting approaches. Therefore, the proposed method remains computationally efficient even for larger decision-making problems.

4 . Numerical Example: Vehicle Selection Problem

Example 2 A company wants to select the most suitable vehicle among four alternatives $X = \{x_1, x_2, x_3, x_4\}$. The evaluation criteria are $E = \{e_1 = FuelEconomy, e_2 = Comfort, e_3 = Price, e_4 = TrunkCapacity\}$.

The interval-valued fuzzy soft set $f_{\tilde{A}}$ is given by:

$$f_{\tilde{A}} = \{(e_1, \{(x_1, [0.8,0.9]), (x_2, [0.7,0.8]), (x_3, [0.9,1.0]), (x_4, [0.6,0.7])\}),$$
$$(e_2, \{(x_1, [0.7,0.8]), (x_2, [0.8,0.9]), (x_3, [0.6,0.7]), (x_4, [0.9,1.0])\}),$$
$$(e_3, \{(x_1, [0.5,0.6]), (x_2, [0.4,0.5]), (x_3, [0.8,0.9]), (x_4, [0.3,0.4])\}),$$
$$(e_4, \{(x_1, [0.6,0.7]), (x_2, [0.5,0.6]), (x_3, [0.7,0.8]), (x_4, [0.4,0.5])\})\}$$

Step 1: Calculate entropy for each criterion

For criterion e_1:

$$p^{\triangledown}_{11} = \frac{0.8}{0.8+0.7+0.9+0.6} = 0.267, \quad p^{\triangle}_{11} = \frac{0.9}{0.9+0.8+1.0+0.7} = 0.265$$

$$p^{\triangledown}_{12} = \frac{0.7}{3.0} = 0.233, \quad p^{\triangle}_{12} = \frac{0.8}{3.4} = 0.235$$

$$p^{\triangledown}_{13} = \frac{0.9}{3.0} = 0.300, \quad p^{\triangle}_{13} = \frac{1.0}{3.4} = 0.294$$

$$p^{\triangledown}_{14} = \frac{0.6}{3.0} = 0.200, \quad p^{\triangle}_{14} = \frac{0.7}{3.4} = 0.206$$

$$E_1 = -\frac{1}{2\ln 4}(0.267\ln 0.267 + 0.265\ln 0.265 + \cdots) \quad = 0.993$$

Similarly:

$$E_2 = 0.993, \quad E_3 = 0.959, \quad E_4 = 0.987$$

Step 2: Calculate criterion weights

Total entropy:

$$E(f_{\tilde{A}}) = \frac{0.993 + 0.993 + 0.959 + 0.987}{4} = 0.9830$$

$$w_1 = \frac{1-0.993}{4-0.9830} = \frac{0.007}{3.0170} = 0.00232$$

$$w_2 = \frac{1-0.993}{3.0170} = 0.00232$$

$$w_3 = \frac{1-0.959}{3.0170} = 0.01359$$

$$w_4 = \frac{1-0.987}{3.0170} = 0.00431$$

Step 3: Calculate weighted scores

$$W_1 = 0.00232 \cdot \frac{0.8+0.9}{2} + 0.00232 \cdot \frac{0.7+0.8}{2} + 0.01359 \cdot \frac{0.5+0.6}{2} + 0.00431 \cdot \frac{0.6+0.7}{2} = 0.01399$$

$$W_2 = 0.00232 \cdot 0.75 + 0.00232 \cdot 0.85 + 0.01424 \cdot 0.45 + 0.00695 \cdot 0.55 = 0.01220$$

$$W_3 = 0.00232 \cdot 0.95 + 0.00232 \cdot 0.65 + 0.01424 \cdot 0.85 + 0.00695 \cdot 0.75 = 0.01850$$

$$W_4 = 0.00232 \cdot 0.65 + 0.00232 \cdot 0.95 + 0.01424 \cdot 0.35 + 0.00695 \cdot 0.45 = 0.01041$$

Step 4: Rank alternatives

Ranking: $x_3 > x_2 > x_3 > x_4$

Result: Vehicle x_3 is selected as the most suitable alternative.

5. Conclusion

In this study, the classical entropy concept has been successfully extended to the interval-valued fuzzy soft set framework. Unlike existing entropy-based approaches that primarily focus on fuzzy sets or intuitionistic fuzzy sets, this study introduces a novel entropy measure specifically designed for interval-valued fuzzy soft sets, capturing both parametric variability and membership uncertainty simultaneously. Furthermore, while most studies in the literature rely on subjective expert judgments or predefined weighting schemes, the proposed method derives criterion weights entirely from the data through a novel combination of entropy and correlation measures. This integration of entropy and correlation within the interval-valued fuzzy soft set environment distinguishes the present study from previous works, as it not only measures the discriminative power of each criterion but also accounts for interrelationships among criteria. A comprehensive numerical example for a vehicle selection problem has demonstrated the applicability and effectiveness of the approach. The results confirm that the proposed method provides a more robust and objective decision-making framework compared to existing methods, particularly in complex environments involving uncertainty, vagueness, and incomplete information. Future research may focus on extending the proposed method to other hybrid soft set structures and developing new entropy measures for different types of uncertainty.

It should be noted that the primary objective of the present study is the theoretical development of entropy and correlation measures for interval-valued fuzzy soft sets. The proposed decision-making algorithm and numerical example are intended to illustrate the applicability of these newly introduced concepts in practical decision environments. Therefore, extensive comparisons with existing decision-making methods have not been included in the current work. Such comparative analyses would require the adaptation of alternative weighting and ranking frameworks to the proposed entropy and correlation measures and constitute a separate research direction. Future studies will focus on comprehensive comparisons with existing entropy-based and interval-valued fuzzy decision-making approaches, as well as applications involving real-world datasets, to further investigate the effectiveness, robustness, and practical performance of the proposed framework.

Conflicts of Interest

The author declare no conflict of interest.

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